Objectives

- Arithmetic Mean
- Standard Deviation
- Correlation Coefficient
- Estimating MTBF
  - Type I Censoring
  - Type II Censoring
- Exponential Distribution
- Reliability Predictions
- Weibull Curves and Intro to Weibull Analysis
- Basic System Reliability
  - Series System
  - Active Parallel Systems
Arithmetic Mean

- The arithmetic mean or simply “mean” is the sum of a group of numbers divided by the number of items in the group.
- In statistics, this is denoted by $\bar{x}$ (pronounced “x bar”)
- Example: What is the arithmetic mean of 24, 37, 16 and 21?

$$\bar{x} = \frac{24 + 37 + 16 + 21}{4} = 24.5$$
Arithmetic Mean

**Example**

\[
\bar{x} = \frac{1}{N} \sum_{i=1}^{N} x_i
\]

\(x_1 = 24\)
\(x_2 = 37\)
\(x_3 = 16\)
\(x_4 = 21\)

\[
\bar{x} = \frac{1}{4} \left( x_1 + x_2 + x_3 + x_4 \right)
\]

\[
\bar{x} = \frac{1}{4} \left( 24 + 37 + 16 + 21 \right)
\]

\[
\bar{x} = \frac{1}{4} \times 98
\]

\[
\bar{x} = \frac{98}{4}
\]

\[
\bar{x} = 24.5
\]
Standard Deviation

- Standard deviation is the measure of statistical dispersion in a set of numbers.
- It is the Root Mean Square (RMS) of the deviation from the arithmetic mean of a group of numbers.
- If the data points are all close to the mean then the Standard Deviation is close to zero.
- If the data points are far from the mean then the standard deviation is far from zero.
- Standard deviation is noted by the lower case Greek letter Sigma (σ)
Standard Deviation

Example

\[ x_1 = 24 \]
\[ x_2 = 37 \]
\[ x_3 = 16 \]
\[ x_4 = 21 \]

For known population size

\[ \sigma = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (x_i - \bar{x})^2} \]

Estimate for unknown population size

\[ \sigma = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (x_i - \bar{x})^2} \]

\[ \sigma = \sqrt{\frac{1}{4} [(24 - 24.5)^2 + (37 - 24.5)^2 + (16 - 24.5)^2 + (21 - 24.5)^2]} \]

\[ \sigma = 7.76208 \]
Correlation Coefficient

- Is the likelihood that 2 sets of numbers are related
  - The closer the correlation value gets to 1.0, the more linear the relationship between the 2 sets of numbers.

- It is based on calculations of slope (m), y-intercept (b) and correlation (r)

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<th>y</th>
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<tr>
<td>6.3</td>
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Slope 0.584
Y-Intercept 1.684
Correlation 0.974
Correlation Coefficient

Using the X & Y values, there is a non-graphical method for calculating slope (m), y intercept (b) and correlation (r).

\[
m = \frac{n \sum (xy) - \sum x \sum y}{n \sum (x^2) - (\sum x)^2}
\]

\[
b = \frac{\sum y - m \sum x}{n}
\]

\[
r = \frac{n \sum (xy) - \sum x \sum y}{\sqrt{n \sum (x^2) - (\sum x)^2} \sqrt{n \sum (y^2) - (\sum y)^2}}
\]
Correlation Coefficient

• Why do I need this? How can I use it?

• Does equipment become more prone to failure or more expensive failures as it ages?
  – Collect some ages and failure rate data and find out?
  – Collect some ages and MTBF and find out?

• Other examples:
  – For a pump, are motor amps and gallons per minute perfectly linear?
Maintenance Costs versus Vibration Analysis (PdM)

Maintenance Costs versus Equipment on PM

Mean Time Between Failures

- MTBF is supposed to be calculated for each individual asset

- Do you calculate it at your plant?
Estimating MTBF

Type I Censoring

- a.k.a. Time/Cycle Truncated Censoring
- Test is halted at a given number of hours.
- Failures during the test are immediately repaired and the test continues

\[ \hat{\Theta} = \frac{nt}{r} \]

Where:
- \( \hat{\Theta} \) = estimate of MTBF
- \( n \) = number of items on test
- \( t \) = total test time per unit
- \( r \) = \# of failures occurring during the test
Estimating MTBF
Type II Censoring

• a.k.a. Failure Truncated Censoring
• Test is halted at a given number of failures
• Failures during the test are immediately repaired and the test continues

\[ \hat{\Theta} = \frac{\sum_{i=1}^{r} y_i + (n - r) y_r}{r} \]

\( \hat{\Theta} \) = estimate of MTBF

\( y_i \) = time to failure \( i_{th} \) item

\( y_r \) = time to failure of the unit at which time is truncated

\( n \) = Total number of assets in test

\( r \) = Total number of failures
When would I use MTBF?

- Good question!

- MTBF can be used to help determine maintenance intervals.
- There is a significant flaw with this.

- What does the M in MTBF stand for?
- What does this implicitly tell you?
Reliability Predictions

- If I know a little bit about the MTBF for a particular asset…

- I can make some predictions about the life of that asset.
Reliability Predictions

- $Q$ is the probability of failure.

- $Q = 1 - R$

- So then $R$ is the probability of not failing
Exponential Distribution

\[ R(t) \]

Time

Reliability

\[ R(t) \]

t
Reliability Predictions

The reliability for a given time \((t)\) during the random failure period can be calculated with the formula:

\[
R(t) = e^{-\lambda t}
\]

Where:
- \(e\) = base of the natural logarithms which is 2.718281828…
- \(\lambda\) = failure rate \((1/MTBF)\)
- \(t\) = time
**e - the base of natural logarithms**

\[
1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \ldots
\]

\[
1 + \frac{1}{1 \cdot 2} + \frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{1 \cdot 2 \cdot 3 \cdot 4} + \ldots
\]

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<th></th>
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</tbody>
</table>

\[
e = 2.71828180114638
\]
Reliability Predictions

Example

A particular pump has a MTBF of 4,000 hours. What is the probability of operating for a period of 1,500 hours without a failure?

\[
\lambda = 0.00025 \text{ or } 1/4,000 \\
t = 1,500 \\
\]

\[
e^{-\lambda t} = e^{-(0.00025)(1,500)} = e^{-0.375} = 0.68728
\]

68.73% Probability exists of operating 1,500 hours without a failure exists when the MTBF = 4,000 hours.

31.27% Probability exists of a failure before operating 1,500 hours.
Reliability Predictions

If the reliability for a given time \( t \) during the random failure period can be calculated with the formula:

\[
R(t) = e^{-\lambda t}
\]

Then what is the equation when I am not in the random failure period? What if I in the infant mortality period or wear-out period?

Then the equation is slightly more difficult…
Overall (Bathtub) Curve

- Infant Mortality
- Random Failure
- Wear Out
Weibull Shapes

Individual Curves

- Bathtub: Pattern A = 4%
- Wear out: Pattern B = 2%
- Fatigue: Pattern C = 5%
- Initial Break-in period: Pattern D = 7%
- Random: Pattern E = 14%
- Infant Mortality: Pattern F = 68%

Age Related = 11%
Random = 89%
Weibull Analysis

\[ R(t) = e^{-\left( \frac{t}{\eta} \right)^\beta} \]
System Reliability

- Rarely do assets work alone
- Typically they are a part of a system
- Systems can many different configurations
  - Series
  - Active Parallel
  - “Hot” Standby Parallel
  - “Warm” Standby Parallel
  - “Cold” Standby Parallel
- Reliability calculations for each of these is slightly different
Series Systems - Reliability

• A system whereby the failure of a single machine shuts down the entire system is said to be a “series designed system”

\[ R_s = R_1 \times R_2 \times R_3 \]

\[ R_s = 0.93 \times 0.91 \times 0.80 = 0.677 \text{ or } 67.7\% \]
Series Systems – Failure Probability

\[ R_s(t) = e^{-\sum \lambda_i t_i} \]
\[ R_s(1500) = e^{-\sum (0.0001+0.00005+0.00001)(1500)} \]
\[ R_s(1500) = e^{-(0.00016)(1500)} \]
\[ R_s(1500) = e^{-(0.24)} \]
\[ R_s(1500) = 0.7866 \]
\[ R_s(1500) = 78.6\% \]
Active Parallel Systems - Reliability

- A system where either machine can carry the full system load and a single failure does not disrupt the system is said to be an “active parallel system”

\[ R_s = R_1 + R_2 - R_1 R_2 \]

\[ R_s = 0.93 + 0.80 - 0.93 \times 0.80 = 0.986 \text{ or } 98.6\% \]
Active Parallel Systems – Failure Probability

\[ R(t) = e^{-\lambda_1 t} + e^{-\lambda_2 t} - e^{-(\lambda_1 + \lambda_2)t} \]

\[ R_{(1500)} = e^{-(0.0001)(1500)} + e^{-(0.0004)(1500)} - e^{-(0.0001+0.0004)(1500)} \]

\[ R_{(1500)} = 0.9372 \]

\[ R_{(1500)} = 93.72\% \]
Questions?

Thanks!

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